Re-Written by Jagdev Singh Dosanjh
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**B.A./B.Sc. 1st Semester (Batch 2024–27/28)**

**Subject: Mathematics (Algebra)**

**Exam Code:** 121201 **Subject Code:** 102689

**Time Allowed:** 3 Hours **Maximum Marks:** 100

**Instructions:** Attempt FIVE questions in all, selecting at least ONE question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

**SECTION-A**

1. (a) Find the rank of the matrix:

$$\begin{bmatrix} 2 & -1 & 0 & 4 \\ 1 & 3 & 5 & -3 \\ 3 & -5 & -5 & 11 \\ 6 & 4 & 10 & 2 \end{bmatrix}$$

(b) Show that vectors (1, 2, -3), (1, -3, 2), and (2, -1, 5) are linearly independent.

2. (a) Solve the system of equations: $x + 2y + z = 1$ $2x + y - z = 0$ $x - y - z = 1$
(b) Prove that every skew-symmetric matrix of odd order has rank less than its order.

**SECTION-B**

3. (a) Determine the eigenvalues and eigenvectors of the matrix:

$$\begin{bmatrix} 1 & 6 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$$

(b) Use Cayley-Hamilton theorem to find the inverse of matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

4. (a) Prove that characteristic roots of a Hermitian matrix are real. (b) Find the quadratic form corresponding to the symmetric matrix:

$$\begin{bmatrix} 0 & a & b & c \\ a & 0 & u & w \\ b & u & 0 & v \\ c & w & v & 0 \end{bmatrix}$$

Jagdevsinghdosanjh@gmail.com
Mobile : 81465-53307

**SECTION-C**

5. (a) Reduce $x^2 + 2y^2 + 2z^2 - 2xy - 2yz + xz$to canonical form. Find the rank and index. (b) Show that every positive definite or semi-definite matrix can be represented as a Gram matrix.

6. (a) Prove that range of values of two congruent quadratic forms are the same. (b) Show that the form $5x^2 + 26y^2 + 10z^2 + 6xy + 4yz + 14zx$ is positive semi-definite and find a non-zero set of values of x, y, z which makes the form zero.

**SECTION-D**

7. (a) Remove the second term from the equation $2x^3 - 9x^2 + 13x - 6 = 0$and hence solve it. (b) Use Cardan's method to solve $28x^3 - 9x^2 + 1 = 0$.

8. (a) Solve by Ferrari's method: $2x^4 + 6x^3 - 3x^2 + 2 = 0$(b) Use Descartes' method to solve $x^4 - 10x^2 - 20x - 16 = 0$

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**B.A./B.Sc. 1st Semester (Old Syllabus, Batch 2023–26)**
**Subject: Physics – Paper A (Mechanics)**
**Exam Code:** 121201 **Subject Code:** 107047

**Time Allowed:** 3 Hours **Maximum Marks:** 75

**Instructions:** Attempt FIVE questions in all, selecting at least ONE question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

**SECTION-A**

1. (a) Define solid angle and give its units. Derive an expression for the solid angle in spherical polar coordinates. (b) Convert Cartesian coordinates (1, 0, 1) into spherical polar coordinates.
2. What are the properties of space and time? Show that homogeneity of space leads to conservation of linear momentum.

**SECTION-B**

3. (a) State and derive Kepler's laws of planetary motion. (b) Define central forces. Prove that a central force is the negative gradient of a scalar potential.
4. Derive the equation of orbit for a particle under inverse square law of force. Explain how orbit shape depends on energy and angular momentum.

**SECTION-C**

5. (a) Define Galilean transformations. Show that length and acceleration are invariant under them, but velocity is not. (b) State the conditions under which Coriolis force on a particle is zero.
6. What is Foucault's pendulum? How does it demonstrate Earth's rotation?

**SECTION-D**

7. (a) What is Rutherford scattering? Derive the expression for Rutherford scattering cross-section for α-particles.
8. (a) What is a gyroscope? Explain precession and derive the expression for precessional angular velocity. (b) Do internal torques affect rotational motion of a rigid body? Briefly explain.

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**B.A./B.Sc. 1st Semester (Old Syllabus, Batch 2023–26)**

**Subject: Mathematics – Paper II (Calculus & Trigonometry)**

**Exam Code:** 121201 **Subject Code:** 107046

**Time Allowed:** 3 Hours **Maximum Marks:** 75

**Instructions:** Attempt FIVE questions in all, selecting at least ONE question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

**SECTION-A**

1. (a) Prove that between any two distinct real numbers, there exists a rational number—and infinitely many. (b) Prove that the limit

$$\lim_{x\to 1} \frac{1}{1-x^5} - \frac{1}{1+x^7}$$

does not exist.

2. (a) Prove that

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

(b) Let

$$f(x) = \{\begin{matrix} ax^2 + bx + 1, & 2 < x < 3 \\ 17 - ax, & x \geq 3 \end{matrix}$$

Determine values of a and b for continuity.

**SECTION-B**

3. (a) State and prove Leibnitz's Theorem. (b) Differentiate: (i) $\tanh^{-1}(\frac{x+1}{x^2-1})$ (ii) $x^2\sqrt{x^2+2} + 2\sinh\ x$

4. (a) State and prove Taylor's Theorem with Lagrange's form of remainder. (b) Find a and b such that

$$\lim_{x\to 0} \frac{x(1 - a\cos\ x) + b\sin\ x}{x} = 3$$

Jagdevsinghdosanjh@gmail.com
Mobile : 81465-53307

**SECTION-C**

5. (a) State and prove De Moivre's Theorem. (b) Solve $(1+z)^n + z^n = 0$, where z is complex.

6. (a) Separate $\cos^{-1}(\cos\theta + i\sin\theta)$ into real and imaginary parts. (b) If $\cosh(u + iv) = x + iy$, prove: (i) $\cosh^2 u - \sinh^2 u = 1$ (ii) $\cos^2 v + \sin^2 v = 1$

**SECTION-D**

7. (a) Prove that $i^i$ is wholly real and find its principal value. Show that its values form a G.P.
(b) Prove:

$$\sin^7\theta = \frac{1}{128}[\cos 7\theta - 7\cos 5\theta + 21\cos 3\theta - 35\cos\theta]$$

8. (a) Using Gregory's series, prove:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

(b) Find the sum of the infinite series:

$$\sin a \sin b + \sin 2a \sin 2b + \sin 3a \sin 3b + \cdots$$